Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

From reference sheet, equation of tangent is $y = tx - at^2$

At D on the directrix, $y = -a$

$$\Rightarrow \quad -a = tx - at^2$$

$$tx = at^2 - a$$

$$x = at - \frac{a}{t}$$

$\therefore D$ is $\left(at - \frac{a}{t}, -a\right)$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	3
• Attempts to eliminate the parameter to find the Cartesian equation of $\mathcal{P}_2$, or equivalent merit	2
• Finds coordinates of R, or equivalent merit	1

Sample answer:

From reference sheet, equation of normal is $x + ty = at^3 + 2at$

At R (above D), $x = at - \dfrac{a}{t}$

$$\Rightarrow \quad at - \frac{a}{t} + ty = at^3 + 2at$$

$$ty = at^3 + at + \frac{a}{t}$$

$$y = at^2 + a + \frac{a}{t^2}$$

$$= a\left(t^2 + 1 + \frac{1}{t^2}\right)$$

$$\therefore R \text{ is } \left(a\left(t - \frac{1}{t}\right),\ a\left(t^2 + 1 + \frac{1}{t^2}\right)\right)$$

Method 1

$$x = a\left(t - \frac{1}{t}\right)$$

$$x^2 = a^2\left(t^2 - 2 + \frac{1}{t^2}\right)$$

$$\frac{x^2}{a^2} = t^2 + \frac{1}{t^2} - 2$$

$$\frac{x^2}{a^2} + 2 = t^2 + \frac{1}{t^2}$$

$$y = a\left(\left(t^2 + \frac{1}{t^2}\right) + 1\right)$$

$$= a\left(\frac{x^2}{a^2} + 2 + 1\right)$$

$$y = \frac{x^2}{a} + 3a$$

$$ay = x^2 + 3a^2$$

Method 2

$$\frac{x}{a} = t - \frac{1}{t}$$

$$y = a\left(t^2 + 1 + \frac{1}{t^2}\right)$$

$$= a\left(\left(t - \frac{1}{t}\right)^2 + 3\right)$$

$$= a\left(\frac{x^2}{a^2} + 3\right) = \frac{x^2}{a} + 3a$$

$$\therefore (y - 3a)a = x^2$$

Locus is $x^2 = a(y - 3a)$.

A parabola! Vertex $(0, 3a)$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$x^2 = a(y - 3a)$$

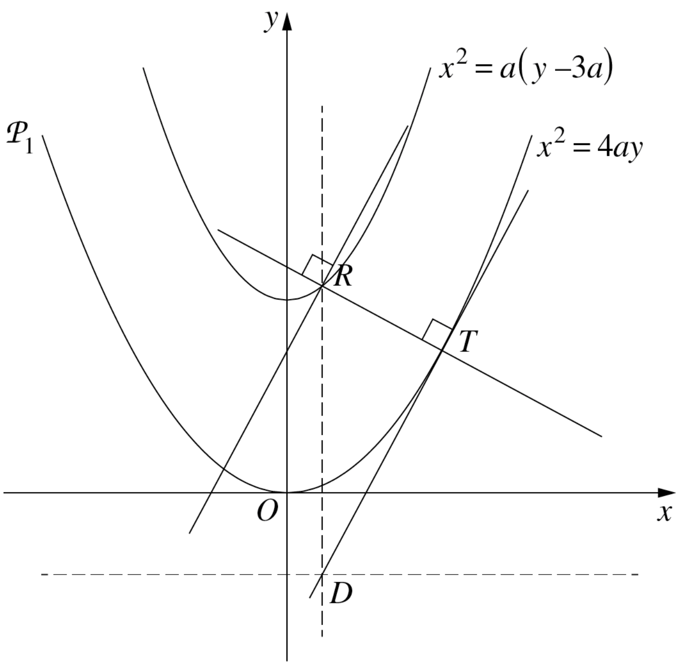
ie $x^2 = 4\left(\frac{a}{4}\right)(y - 3a)$

$\therefore$ Focal length $\frac{a}{4}$

Question 14 (c) (iv)

Criteria	Marks
• Provides correct solution	2
• Finds slope of $\mathcal{P}_2$ at R, or equivalent merit	1

Sample answer:



$T(2at,\ at^2)$

$R\left(at - \frac{a}{t},\ a\left(t^2 + 1 + \frac{1}{t^2}\right)\right)$

A common normal will occur when the tangents at T and R are parallel.

Now gradient at T is t

For R, $\quad \frac{x^2}{a} = y - 3a$

Question 14 (c) (iv) continued

$$y = \frac{x^2}{a} + 3a$$

$$y' = \frac{2x}{a}$$

So $\quad y' = \dfrac{2\left(at - \dfrac{a}{t}\right)}{a}$, since $x = at - \dfrac{a}{t}$,

$$= 2\left(t - \frac{1}{t}\right)$$

So gradient at R is $2t - \dfrac{2}{t}$

For tangents to be parallel, $t = 2t - \dfrac{2}{t}$

$$t^2 = 2t^2 - 2$$

$$2 = t^2$$

$$t = \pm\sqrt{2}$$

2016 HSC Mathematics Extension 1
Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	7.1	H5, HE7
2	1	16.2E	PE3
3	1	5.7E	P3, PE6
4	1	2.10E, 2.9E	PE3
5	1	13.6E	HE6
6	1	15.2E, 5.9E	HE4
7	1	14.3, 14.4E	HE5
8	1	18.1E	PE3
9	1	10.1, 10.4	H6
10	1	16.1 E, 16.3E	PE3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	15.1E	HE4
11 (b)	3	11.5E	HE6
11 (c)	2	15.5E	HE4
11 (d)	2	5.7E, 13.4E	HE7
11 (e)	3	1.4E	PE3
11 (f) (i)	1	3.2, 18.2E	HE3
11 (f) (ii)	2	18.2E	HE3
12 (a) (i)	1	2.3	P2, PE6
12 (a) (ii)	1	14.1E	HE5
12 (a) (iii)	2	14.1E	HE5
12 (a) (iv)	2	14.1E	HE5
12 (b) (i)	3	14.2E	HE3
12 (b) (ii)	2	14.2E	HE3
12 (c) (i)	2	6.2, 10.7, 13.5	H5, HE4
12 (c) (ii)	2	16.4E	HE7
13 (a) (i)	2	14.4E	HE3
13 (a) (ii)	2	14.4E	HE3

Question	Marks	Content	Syllabus outcomes
13 (b) (i)	2	14.3E	HE3
13 (b) (ii)	2	14.3E	HE3
13 (b) (iii)	2	14.3E	HE3
13 (b) (iv)	1	14.3E	HE3
13 (c) (i)	2	2.10E, 2.7E	PE3
13 (c) (ii)	2	2.10E, 2.8E	PE3
14 (a) (i)	1	1.3	P4, PE6
14 (a) (ii)	3	7.4E	HE2
14 (b) (i)	1	17.1E, 17.3E	HE7
14 (b) (ii)	1	8.7, 17.1E, 17.3E	HE4, HE7
14 (b) (iii)	2	17.1E, 17.3E	HE7
14 (c) (i)	1	9.6E	PE3
14 (c) (ii)	3	9.6E	PE3
14 (c) (iii)	1	9.6E	PE3
14 (c) (iv)	2	10.7, 9.6E	HE7