

2017 HSC Mathematics Extension 1

Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	B
3	B
4	C
5	D
6	D
7	A
8	C
9	C
10	B

Section II

Question 11 (a)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{aligned}
 x \text{ value of } P &= \frac{2 \times 1 + 3 \times -4}{2 + 3} \\
 &= -\frac{10}{5} \\
 &= -2
 \end{aligned}$$

Question 11 (b)

Criteria	Marks
• Provides correct solution	2
• Attempts to use the chain rule, or equivalent merit	1

Sample answer:

$$\text{Let } y = \tan^{-1}(x^3)$$

$$\begin{aligned}
 y' &= \frac{1}{1 + (x^3)^2} \times 3x^2 \\
 &= \frac{3x^2}{1 + x^6}
 \end{aligned}$$

Question 11 (c)

Criteria	Marks
• Provides correct solution	3
• Correctly identifies 1 and -1 as important, or equivalent merit	2
• Attempts to deal with the denominator, or equivalent merit	1

Sample answer:

$$\frac{2x}{x+1} > 1$$

Multiply both sides by $(x+1)^2$

$$2x(x+1) > (x+1)^2$$

$$2x^2 + 2x > x^2 + 2x + 1$$

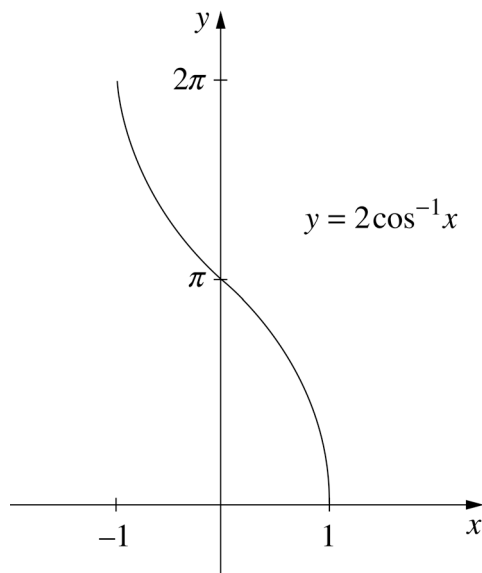
$$x^2 > 1$$

$$\therefore x > 1 \text{ or } x < -1$$

Question 11 (d)

Criteria	Marks
• Provides correct sketch	2
• Indicates correct range, or equivalent merit	1

Sample answer:



Question 11 (e)

Criteria	Marks
• Provides correct solution	3
• Provides a correct primitive, or equivalent merit	2
• Attempts to use given substitution, or equivalent merit	1

Sample answer:

$$\int_0^3 \frac{x}{\sqrt{x+1}} dx \quad \begin{array}{l} x = u^2 - 1 \\ \frac{dx}{du} = 2u \\ dx = 2u du \end{array}$$

$$\begin{array}{lll} \text{when } x=0 & u^2 - 1 = 0 & u = 1 \\ \text{when } x=3 & u^2 - 1 = 3 & u = 2 \end{array}$$

$$\begin{aligned} \int_0^3 \frac{x}{\sqrt{x+1}} dx &= \int_1^2 \frac{u^2 - 1}{u} \cdot 2u du \\ &= 2 \int_1^2 (u^2 - 1) du \\ &= 2 \left[\frac{u^3}{3} - u \right]_1^2 \\ &= 2 \left[\left(\frac{8}{3} - 2 \right) - \left(\frac{1}{3} - 1 \right) \right] \\ &= \frac{8}{3} \end{aligned}$$

Question 11 (f)

Criteria	Marks
• Provides correct primitive	1

Sample answer:

$$\begin{aligned} \int \sin^2 x \cos x dx \\ = \frac{\sin^3 x}{3} + c \end{aligned}$$

Question 11 (g) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\binom{8}{3} \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^5$$

Question 11 (g) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\left(\frac{4}{5}\right)^8$$

Question 11 (g) (iii)

Criteria	Marks
• Provides correct answer	1

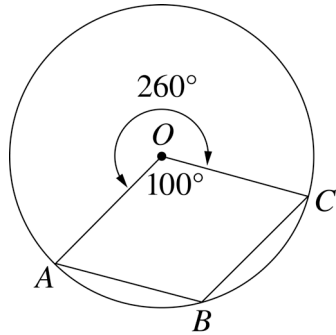
Sample answer:

$$1 - \left(\frac{4}{5}\right)^8$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	2
• Attempts to relate two angles in the diagram, or equivalent merit	1

Sample answer:

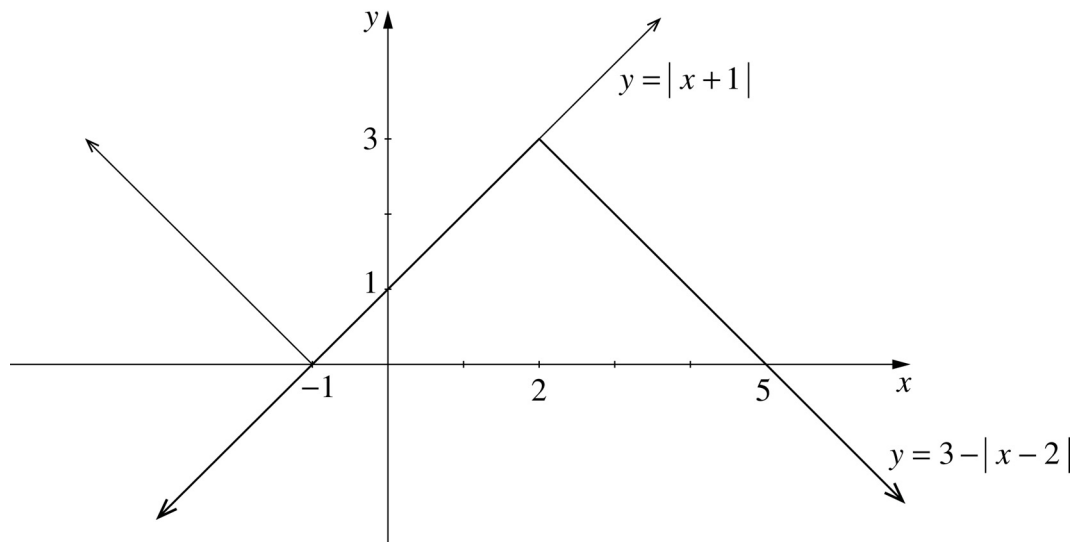


$$\begin{aligned}
 \text{Reflex } \angle AOC &= 360^\circ - 100^\circ && \text{(angle of revolution)} \\
 &= 260^\circ \\
 \angle ABC &= \frac{1}{2} \text{ reflex } \angle AOC && \text{(angle at centre equals twice the angle at} \\
 &&& \text{circumference standing in the same arc)} \\
 &= \frac{1}{2} (260^\circ) \\
 &= 130^\circ
 \end{aligned}$$

Question 12 (b) (i)

Criteria	Marks
• Provides correct sketch	3
• Provides correct sketch of $y = 3 - x - 2 $, or equivalent merit	2
• Provides correct sketch of $y = x + 1 $, or equivalent merit	1

Sample answer:



Question 12 (b) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\therefore -1 \leq x \leq 2$$

when $|x + 1| = 3 - |x - 2|$ the graphs have points in common.
ie $|x + 1| + |x - 2| = 3$

Question 12 (c) (i)

Criteria	Marks
• Provides correct solution	3
• Equates the volumes in the correct ratio and attempts to evaluate an integral, or equivalent merit	2
• Provides correct integral for the volume of one solid, or equivalent merit	1

Sample answer:

$$V_1 = \pi \int_h^1 y^2 dx$$

$$= \pi \int_h^1 1 - x^2 dx$$

$$= \pi \left[x - \frac{x^3}{3} \right]_h^1$$

$$= \pi \left[1 - \frac{1}{3} \right] - \pi \left[h - \frac{h^3}{3} \right]$$

$$= \pi \left[\frac{2}{3} - h + \frac{h^3}{3} \right]$$

$$V_2 = \pi \int_{-1}^h 1 - x^2 dx$$

$$= \pi \left[x - \frac{x^3}{3} \right]_{-1}^h$$

$$= \pi \left[h - \frac{h^3}{3} \right] - \pi \left[-1 - \frac{-1}{3} \right]$$

$$= \pi \left[h - \frac{h^3}{3} + \frac{2}{3} \right]$$

Ratio $V_2 : V_1 = 2 : 1 \quad \therefore V_2 = 2V_1$

$$h - \frac{h^3}{3} + \frac{2}{3} = 2 \left(\frac{2}{3} - h + \frac{h^3}{3} \right)$$

$$= \frac{4}{3} - 2h + \frac{2h^3}{3}$$

$$\therefore 3h - h^3 + 2 = 4 - 6h + 2h^3$$

$$\therefore 3h^3 - 9h + 2 = 0$$

Question 12 (c) (ii)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\text{Let } f(h) = 3h^3 - 9h + 2 \quad h_1 = 0$$

$$h_2 = h_1 - \frac{f(h_1)}{f'(h_1)} = 0 - \frac{f(0)}{f'(0)}$$

$$f(h_1) = f(0) = 2$$

$$f'(h) = 9h^2 - 9$$

$$\begin{aligned}\therefore f'(0) &= 9(0)^2 - 9 \\ &= -9\end{aligned}$$

$$\begin{aligned}\therefore h_2 &= 0 - \frac{2}{-9} \\ &= \frac{2}{9}\end{aligned}$$

Question 12 (d)

Criteria	Marks
• Provides correct solution	3
• Attempts to use $\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$, or equivalent merit	2
• Correctly finds $\frac{dt}{dx}$, or equivalent merit	1

Sample answer:

$$t = 4 - e^{-2x}$$

$$\frac{dt}{dx} = +2e^{-2x}$$

$$\therefore \frac{dx}{dt} = \frac{1}{2}e^{2x}$$

$$v = \frac{1}{2}e^{2x}$$

$$v^2 = \frac{1}{4}e^{4x}$$

$$\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$= \frac{d}{dx}\left(\frac{1}{8}e^{4x}\right)$$

$$= \frac{4}{8}e^{4x}$$

$$\ddot{x} = \frac{1}{2}e^{4x}$$

Question 12 (e)

Criteria	Marks
• Provides correct solution	2
• Correctly uses double angle result, or equivalent merit	1

Sample answer:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2} \\ &= \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} \text{ since } \cos 2x = 1 - 2\sin^2 x \\ &= 2 \lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} \\ &= 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \\ &= 2 \times 1^2 \\ &= 2 \end{aligned}$$

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Correctly finds the value of n^2 , or equivalent merit	2
• Correctly uses $v^2 = n^2(a^2 - x^2)$, or equivalent merit	1

Sample answer:

$$v^2 = n^2(a^2 - x^2)$$

$$\therefore 4^2 = n^2 a^2 - 2^2 n^2 \text{ ——— ①}$$

$$\text{and } 3^2 = n^2 a^2 - 5^2 n^2 \text{ ——— ②}$$

$$\text{①-② } \therefore 7 = -4n^2 + 25n^2$$

$$21n^2 = 7$$

$$n^2 = \frac{1}{3}$$

$$n = \frac{1}{\sqrt{3}}, \quad n > 0$$

$$\text{Period, } T = \frac{2\pi}{n}$$

$$= \frac{2\pi}{\frac{1}{\sqrt{3}}}$$

$$= 2\pi\sqrt{3}$$

Question 13 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Correctly applies binomial theorem to one binomial expression, or equivalent merit	1

Sample answer:

n is even

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{r}x^r + \dots + \binom{n}{n}x^n$$

$$(1-x)^n = \binom{n}{0} - \binom{n}{1}x + \binom{n}{2}x^2 + \dots + (-1)^r \binom{n}{r}x^r + \dots + \binom{n}{n}x^n$$

$$(1+x)^n + (1-x)^n = 2\binom{n}{0} + 2\binom{n}{2}x^2 + \dots + 2\binom{n}{n}x^n$$

(odd terms have different sign and so cancel)

$$(1+x)^n + (1-x)^n = 2\left[\binom{n}{0} + \binom{n}{2}x^2 + \binom{n}{4}x^4 + \dots + \binom{n}{n}x^n\right]$$

Question 13 (b) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Differentiating with respect to x .

$$n(1+x)^{n-1} - n(1-x)^{n-1} = 2\left[2\binom{n}{2}x + 4\binom{n}{4}x^3 + \dots + n\binom{n}{n}x^{n-1}\right]$$

$$n((1+x)^{n-1} - (1-x)^{n-1}) = 2\left[2\binom{n}{2}x + 4\binom{n}{4}x^3 + \dots + n\binom{n}{n}x^{n-1}\right]$$

Question 13 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Uses $x = 1$, or equivalent merit	1

Sample answer:

Let $x = 1$

$$n(2^{n-1} - 0) = 2 \left[2 \binom{n}{2} + 4 \binom{n}{4} + \dots + n \binom{n}{n} \right]$$

$$n 2^{n-2} = 2 \binom{n}{2} + 4 \binom{n}{4} + \dots + n \binom{n}{n}$$

$$n 2^{n-3} = \binom{n}{2} + 2 \binom{n}{4} + 3 \binom{n}{6} + \dots + \frac{n}{2} \binom{n}{n}$$

Question 13 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Attempts to solve $y = 0$, or equivalent merit	1

Sample answer:

$$\text{Set } 0 = y = Vt \sin \theta - \frac{1}{2}gt^2$$

$$t = 0 \quad \text{or} \quad V \sin \theta = \frac{1}{2}gt$$

$$t = \frac{2V \sin \theta}{g}$$

At this value

$$x = V \left(\frac{2V \sin \theta}{g} \right) \cos \theta$$

$$= \frac{V^2}{g} \sin 2\theta$$

Question 13 (c) (ii)

Criteria	Marks
• Provides correct solution	1

Sample answer:

$$\text{Range} = \frac{V^2}{g} \sin 2\theta$$

$$\text{If } v^2 < 100g, \text{ then range } \frac{100g \sin 2\theta}{g}$$

$$\text{ie } \text{range} < 100 \sin 2\theta$$

$$\text{since } \sin 2\theta \leq 1$$

$$\text{then } \text{range} < 100$$

Question 13 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Provides an inequality in $\sin 2\theta$, or equivalent merit	1

Sample answer:

$$\text{If } V^2 = 200g, \text{ then range} = \frac{200g \sin 2\theta}{g} \geq 100$$

$$\text{ie } 200 \sin 2\theta \geq 100$$

$$\sin 2\theta \geq \frac{1}{2}$$

$$\text{Now, } 0 \leq \theta \leq \frac{\pi}{2} \text{ or } 0 \leq 2\theta \leq \pi$$

$$\frac{\pi}{6} \leq 2\theta \leq \frac{5\pi}{6}$$

$$\frac{\pi}{12} \leq \theta \leq \frac{5\pi}{12}$$

Question 13 (c) (iv)

Criteria	Marks
• Provides correct solution	2
• Obtains correct expression for the maximum height, or equivalent merit	1

Sample answer:

Maximum height occurs when

$$0 = \frac{dy}{dt} = V \sin \theta - gt$$

$$t = \frac{V}{g} \sin \theta$$

At this time

$$y = \frac{V^2}{g} \sin^2 \theta - \frac{1}{2} g \frac{V^2}{g^2} \sin^2 \theta$$

$$= \frac{V^2 \sin^2 \theta}{g} - \frac{V^2 \sin^2 \theta}{2g}$$

$$= 100 \sin^2 \theta \quad \left(\frac{V^2}{g} = 200 \right)$$

For $0 \leq \theta \leq \frac{\pi}{2}$ $\sin^2 \theta$ is an increasing function,

so maximum occurs when $\theta = \frac{5\pi}{12}$

\therefore greatest height is $100 \sin^2 \left(\frac{5\pi}{12} \right)$

Question 14 (a)

Criteria	Marks
• Provides correct solution	3
• Correctly shows $P(n) \Rightarrow P(n+1)$, or equivalent merit	2
• Establishes result for $n = 1$, or equivalent merit	1

Sample answer:

Let $P(n)$ be the given proposition.

$P(1)$ is true since $8^3 + 6 = 518 = 7 \times 74$ which is divisible by 7.

Let k be an integer for which $P(k)$ is true.

That is $8^{2k+1} + 6^{2k-1} = 7m$, for some integer m .

$$\begin{aligned}
 &\text{Consider } 8^{2(k+1)+1} + 6^{2(k+1)-1} \\
 &= 8^2 \times 8^{2k+1} + 6^2 \times 6^{2k-1} \\
 &= 8^2 \times 8^{2k+1} + 8^2 \times 6^{2k-1} - 8^2 \times 6^{2k-1} + 6^2 \times 6^{2k-1} \\
 &= 8^2(8^{2k+1} + 6^{2k-1}) + (6^2 - 8^2)6^{2k-1} \\
 &= 64 \times 7m - 28 \times 6^{2k-1} \\
 &= 7(64m - 4 \times 6^{2k-1})
 \end{aligned}$$

which is divisible by 7, since m and k are integers and $64m - 4 \times 6^{2k-1}$ is an integer.

$\therefore P(k+1)$ is also true

$\therefore P(n)$ is true for all $n \geq 1$ by induction.

Question 14 (b) (i)

Criteria	Marks
• Provides correct solution	2
• Obtains equation of tangent at P , or equivalent merit	1

Sample answer:

The equation of the tangent at P is

$$y = px - p^2$$

substituting into $x^2 = -4ay$

$$-\frac{x^2}{4a} = px - p^2$$

$$\therefore x^2 + 4apx - 4ap^2 = 0$$

Question 14 (b) (ii)

Criteria	Marks
• Provides correct coordinates	2
• Correctly verifies one coordinate	1

Sample answer:

The sum of the roots of the equation is $-4ap$.

x -coordinate of M is average of the roots

$\therefore x$ -coordinate of M is $-2ap$.

Substituting into the tangent,

$$y = p \times -2ap - p^2$$

$$= -p^2(2a + 1)$$

$\therefore M$ is the point $(-2ap, -p^2(2a + 1))$

Question 14 (b) (iii)

Criteria	Marks
• Provides correct solution	2
• Eliminates the parameter, or equivalent merit	1

Sample answer:

Let $x = -2ap$, $y = -p^2(2a + 1)$

$$\begin{aligned} \therefore y &= -\left(\frac{x}{2a}\right)^2(2a + 1) \\ &= -x^2 \frac{(2a + 1)}{4a^2} \end{aligned}$$

so $x^2 = -\frac{4a^2}{2a + 1}y$

Hence $\frac{a^2}{2a + 1} = 1$

ie $a^2 - 2a - 1 = 0$

$$\therefore a = 1 \pm \sqrt{2}$$

$$\therefore a = 1 + \sqrt{2}, \text{ since } a > 0$$

Question 14 (c) (i)

Criteria	Marks
• Provides correct solution	2
• Correctly applies the product rule, or equivalent merit	1

Sample answer:

$$\begin{aligned} \frac{d}{dt}(F(t)e^{0.4t}) &= F'(t)e^{0.4t} + 0.4F(t)e^{0.4t} \\ &= e^{0.4t}(50e^{-0.5t} - 0.4F(t)) + 0.4F(t)e^{0.4t} \\ &= 50e^{-0.1t} \end{aligned}$$

Question 14 (c) (ii)

Criteria	Marks
• Provides correct solution	2
• Correctly integrates the expression given in part (i), or equivalent merit	1

Sample answer:

Integrating,

$$\begin{aligned}
 F(t)e^{0.4t} &= \int 50e^{-0.1t} dt \\
 &= -500e^{-0.1t} + c
 \end{aligned}$$

Now $F(t) = 0$ when $t = 0 \therefore c = +500$

$$\therefore F(t)e^{0.4t} = 500(1 - e^{-0.1t})$$

$$F(t) = 500(e^{-0.4t} - e^{-0.5t})$$

Question 14 (c) (iii)

Criteria	Marks
• Provides correct solution	2
• Makes substantial progress	1

Sample answer:

$$F'(t) = 500(-0.4e^{-0.4t} + 0.5e^{-0.5t})$$

$$= 0 \text{ for a maximum}$$

$$\therefore 0.4e^{-0.4t} = 0.5e^{-0.5t}$$

$$\therefore \frac{5}{4} = e^{0.1t}$$

$$t = 10 \ln \frac{5}{4}$$

$$(\approx 2.23 \text{ hours})$$

Alternative

$$F'(t) = 50e^{-0.5t} - 0.4F(t)$$

$$= 50e^{-0.5t} - 0.4 \left[500(e^{-0.4t} - e^{-0.5t}) \right]$$

$$= 250e^{-0.5t} - 200e^{-0.4t}$$

For maximum $F'(t) = 0$

$$\therefore 250e^{-0.5t} = 200e^{-0.4t}$$

$$\frac{5}{4} = \frac{e^{-0.4t}}{e^{-0.5t}}$$

$$= e^{0.1t}$$

$$\therefore 0.1t = \ln\left(\frac{5}{4}\right)$$

$$t = 10 \ln\left(\frac{5}{4}\right)$$

2017 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	16.2	PE3
2	1	12.1	HE7
3	1	2.10	PE3
4	1	5.9	PE2
5	1	10.5	HE7
6	1	9.6	PE3
7	1	15.3	HE4
8	1	14.1	HE4
9	1	17.3	HE3
10	1	18.1	HE3

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	1	6.7	PE6
11 (b)	2	8.8, 15.5	HE4
11 (c)	3	1.4	PE3
11 (d)	2	15.3	HE4
11 (e)	3	11.5	HE6
11 (f)	1	13.6E	HE6
11 (g) (i)	1	18.2	HE3
11 (g) (ii)	1	18.2	HE3
11 (g) (iii)	1	18.2	HE3
12 (a)	2	2.8	PE3
12 (b) (i)	3	1.4, 1.2	PE6
12 (b) (ii)	1	1.4, 1.2	PE2
12 (c) (i)	3	11.4	H8, HE7
12 (c) (ii)	1	16.4	HE7
12 (d)	3	14.3	HE5
12 (e)	2	5.7, 8.2, 13.4E	HE7
13 (a)	3	14.4	HE3
13 (b) (i)	2	17.1, 17.3	HE7
13 (b) (ii)	1	17.1, 17.3	HE7
13 (b) (iii)	2	17.1, 17.3	HE7
13 (c) (i)	2	14.3	HE3
13 (c) (ii)	1	14.3	HE3
13 (c) (iii)	2	14.3	HE3

Question	Marks	Content	Syllabus outcomes
13 (c) (iv)	2	14.3	HE3
14 (a)	3	7.4	HE2
14 (b) (i)	2	9.6	PE3, PE4
14 (b) (ii)	2	9.6	PE3, PE4
14 (b) (iii)	2	9.6	PE3, PE4
14 (c) (i)	2	8.8, 11.2, 12.5	HE3
14 (c) (ii)	2	12.5	HE3
14 (c) (iii)	2	10.6, 12.5	HE3