## Question 1

(a) $P(-3)=(-3)^3=-27$.

(b) $\dfrac{-3}{\sqrt{1-9x^2}}$.

(c) $\displaystyle\int_{-1}^{1}\frac{1}{\sqrt{4-x^2}}=\left[\sin^{-1}\frac{x}{2}\right]_{-1}^{1}=2\times\frac{\pi}{6}=\frac{\pi}{3}$.

(d) ${}^{12}C_4 2^8 3^4$.

(e) $\left[\dfrac{\sin^3\theta}{3}\right]_0^{\frac{\pi}{4}}=\dfrac{1}{3}\times\left(\dfrac{\sqrt{2}}{2}\right)^3=\dfrac{2\sqrt{2}}{24}$.

(f) $(x-3)(5-x)>0, \therefore 3<x<5$.

## Question 2

(a) $u=\ln x$, $du=\dfrac{1}{x}dx.$

When $x=e, u=1$; when $x=e^2, u=2.$

$\displaystyle\int_1^2\frac{1}{u^2}du=\left[\frac{-1}{u}\right]_1^2=1-\frac{1}{2}=\frac{1}{2}.$

(b) $\dfrac{1}{2}v^2=\dfrac{x^2}{2}+4x+C.$

When $x=1, v=0, \therefore C=-\dfrac{9}{2}.$

$\therefore v^2=x^2+8x-9.$

When $x=2, v^2=11, \therefore$ Speed $=\sqrt{11}$ m/s.

(c) $\sum\alpha=-2+3+\alpha=\dfrac{-16}{a}.$

$1+\alpha=-\dfrac{16}{a}$ (1)

$\prod\alpha=-6\alpha=\dfrac{120}{a}. \therefore a=-\dfrac{20}{\alpha}.$

From (1), $1+\alpha=\dfrac{16}{20}\alpha=\dfrac{4}{5}\alpha.$

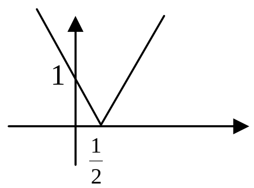
$1=-\dfrac{1}{5}\alpha. \therefore \alpha=-5.$

(d) $f'(x)=\sec^2 x-\dfrac{1}{x}.$

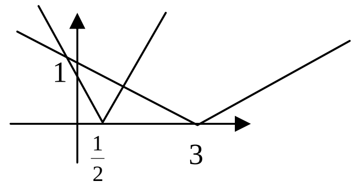
$x_1=4-\dfrac{\tan 4-\ln 4}{\sec^2 4-\frac{1}{4}}=4.11$.

## Question 3

(a)




(b)




From the graph, $|2x-1|\le|x-3|$ for $\pm(2x-1)\le -x+3$.

$2x-1\le -x+3$ gives $3x\le 4, \therefore x\le\dfrac{4}{3}.$

$-2x+1\le -x+3$ gives $x\ge -2.$

$\therefore -2\le x\le\dfrac{4}{3}.$

(c) (i) $\tan\theta=\dfrac{x}{\ell}.$

$\theta=\tan^{-1}\dfrac{x}{\ell}.$

$\dfrac{d\theta}{dt}=\dfrac{d\theta}{dx}\dfrac{dx}{dt}=\dfrac{1}{\ell}\dfrac{1}{1+\frac{x^2}{\ell^2}}\dfrac{dx}{dt}=\dfrac{\ell}{\ell^2+x^2}\times v=\dfrac{v\ell}{\ell^2+x^2}.$

(ii) Given that $v$ and $\ell$ are constant, $\dfrac{d\theta}{dt}$ is the reciprocal of $\ell^2+x^2$, $\therefore$ It is maximum $\ell^2+x^2$ is minimum, i.e. when $x=0$.

$\therefore$ The maximum value of $\dfrac{d\theta}{dt}$ is $\dfrac{v\ell}{\ell^2}=\dfrac{v}{\ell}$.

(iii) $\dfrac{d\theta}{dt}=\dfrac{v}{4\ell}$ gives $\dfrac{v\ell}{\ell^2+x^2}=\dfrac{v}{4\ell}.$

$4\ell^2=\ell^2+x^2.$

$3\ell^2=x^2.$

$\dfrac{x}{\ell}=\pm\sqrt{3}.$

$\tan\theta=\pm\sqrt{3}.$

$\theta=\pm\dfrac{\pi}{3}.$

## Question 4

(a) (i) $T=190-185e^{-kt}$.

When $t=0$, $T=190°-185°=5°$.

$\dfrac{dT}{dt}=185ke^{-ky}=-k(190-T)$.

$\therefore$ It satisfies both the equation and the initial condition.

(ii) When $t=1$, $T=29$: $29=190-185e^{-k}$.

$185e^{-k}=161.$

$-k=\ln\dfrac{161}{185}=-0.1390.$

$k=0.1390.$

When $T = 80, 80 = 190 - 185e^{-0.1390t}$.

$185e^{-0.1390t} = 190 - 80 = 110.$

$-0.1390t = \ln\frac{110}{185}.$

$t = \frac{\ln\frac{110}{185}}{-0.1390} = 3.74.$

3.74 hours = 3 hours 44 minutes.

$\therefore$ The turkey will be cooked at 12:44 pm.

(b) (i) 7! = 5040, (ii) $\frac{8!}{2!} = 20160$.

(c) (i) Gradient of $QO$ = $\frac{aq^2}{2aq} = \frac{q}{2}$.

Gradient of the tangent at $P$ = $\frac{2ap}{2a} = p$.

These two lines are perpendicular, $\therefore \frac{q}{2}p = -1, \therefore pq = -2$.

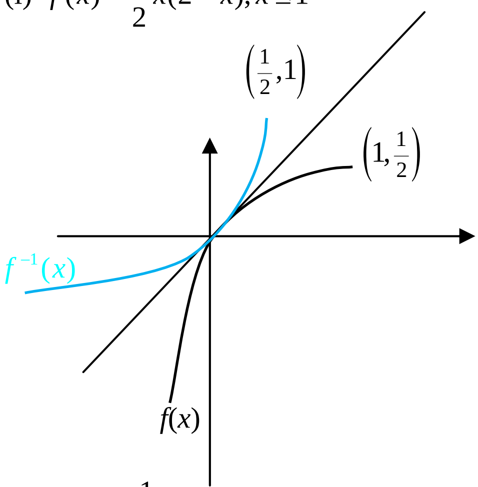
(ii) The gradient of $PO$ is $\frac{p}{2}$ and the gradient of the tangent at $Q$ is $q$, and given $pq = -2$, $\therefore$ PO is perpendicular to the tangent at $Q$. $\therefore \angle PLQ = 90°$.

(iii) $\angle PLQ = \angle PKQ = 90°$, $\therefore PQLK$ is a semicircle on the diameter $PQ$.

If $M$ is the midpoint of $PQ$, $M$ is the centre. $\therefore ML = MK$ = radius.

## Question 5

(a) (i) $f(x) = \frac{1}{2}x(2-x), x \le 1$




(ii) $f: y = x - \frac{1}{2}x^2$.

$f^{-1}: x = y - \frac{1}{2}y^2$.

$y^2 - 2y + 2x = 0.$

$(y-1)^2 = 1 - 2x.$

$y = 1 \pm \sqrt{1-2x}$. Take $y = 1 - \sqrt{1-2x}$ so that $y \le 1$.

(c) $f^{-1}\left(\frac{3}{8}\right) = 1 - \sqrt{1 - \frac{3}{4}} = 1 - \frac{1}{2} = \frac{1}{2}$.

(b) From $v^2 = n^2(A^2 - x^2)$, when $x = 0, v = 2$, $\therefore 4 = n^2A^2$.

From $a = -n^2x$, when $x = A, |a| = 6, \therefore 6 = n^2A$.

$\frac{4}{6} = A, \therefore A = \frac{2}{3}$ m.

$4 = n^2\frac{4}{9}, \therefore n^2 = 9, \therefore n = 3.$

Period $T = \frac{2\pi}{3}$ s.

(c) $\angle PLK = \angle PQM$ (in a cyclic quadrilateral, interior angle = opposite exterior angle)

$\angle PQM = \angle TPM$ (angles in alternate segments are equal)

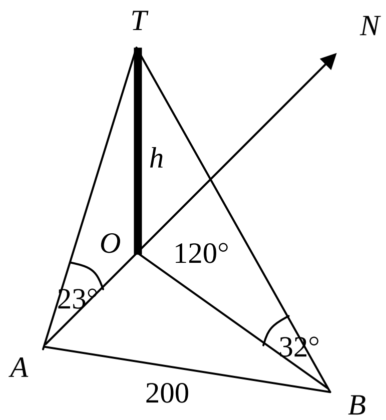
$\angle TPM = \angle LPK$ (vertically opposite angles).

$\therefore \angle PLK = \angle LPK$.

$\therefore \Delta PKL$ is isosceles.

## Question 6

(a) (i)




(ii) $\tan 23° = \frac{h}{OA}, \therefore OA = h\cot 23°.$

$\tan 32° = \frac{h}{OB}, \therefore OB = h\cot 32°.$

$AB^2 = OA^2 + OB^2 - 2OA.OB.\cos(180-120)°.$

$200^2 = h^2\left(\cot^2 23° + \cot^2 32° - \cot 23°\cot 32°\right)$

$h = \frac{200}{\sqrt{\cot^2 23° + \cot^2 32° - \cot 23°\cot 32°}} = 96$ m.

(b) $3\sin\theta - 4\sin^3\theta + \sin 2\theta = \sin\theta$.

$2\sin\theta - 4\sin^3\theta + 2\sin\theta\cos\theta = 0.$

$\sin\theta - 2\sin^3\theta + \sin\theta\cos\theta = 0.$

$\sin\theta\left(1 - 2\sin^2\theta + \cos\theta\right) = 0.$

$\sin\theta\left(1 - 2(1-\cos^2\theta) + \cos\theta\right) = 0.$

$\sin\theta\left(2\cos^2\theta + \cos\theta - 1\right) = 0.$

$\sin\theta(\cos\theta+1)(2\cos\theta-1)=0.$

$\sin\theta=0, \cos\theta=-1, \cos\theta=\frac{1}{2}.$

$\therefore \theta=0,\frac{\pi}{3},\pi,\frac{5\pi}{3},2\pi.$

(c) (i)

$(1+x)^{p+q}={}^{p+q}C_0+{}^{p+q}C_1x+{}^{p+q}Cx^2+\ldots+{}^{p+q}C_{p+q}x^{p+q}.$

$\therefore$ The term independent of $x$ in $\frac{(1+x)^{p+q}}{x^q}$ is ${}^{p+q}C_q$.

(ii) The constant term in the RHS is the sum of the product of each of the following pairs:

| | |
|---|---|
| $\binom{p}{0}x^0$ | $\binom{q}{0}\frac{1}{x^0}$ |
| $\binom{p}{1}x^1$ | $\binom{q}{1}\frac{1}{x}$ |
| $\binom{p}{2}x^2$ | $\binom{q}{2}\frac{1}{x^2}$ |
| … | … |
| $\binom{p}{p}x^p$ | $\binom{q}{p}\frac{1}{x^p}$ |

$1+\binom{p}{1}\binom{q}{1}+\binom{p}{2}\binom{q}{2}+\ldots+\binom{p}{p}\binom{q}{p}$ is the coefficient of the constant term in the expansion of $(1+x)^p\left(1+\frac{1}{x}\right)^q$.

$\therefore$ Its simpler expression is ${}^{p+q}C_q$.

## Question 7

(a) When $y=h,\ h=Vt\sin\theta-\frac{1}{2}gt^2$.

$gt^2-2V\sin\theta t+2h=0.$

$\sum\alpha=t_1+t_2=\frac{2V\sin\theta}{g}$ and $\prod\alpha=t_1t_2=\frac{2h}{g}.$

(b) $\tan\alpha+\tan\beta=\frac{h}{V\cos\theta}\left(\frac{1}{t_1}+\frac{1}{t_2}\right).$

$=\frac{h}{V\cos\theta}\times\frac{t_1+t_2}{t_1t_2}$

$=\frac{h}{V\cos\theta}\times\frac{\frac{2V\sin\theta}{g}}{\frac{2h}{g}}$

$=\tan\theta.$

(c) $\tan\alpha\tan\beta=\frac{h^2}{V^2\cos^2\theta}\left(\frac{1}{t_1t_2}\right).$

$=\frac{h^2}{V^2\cos^2\theta}\times\frac{g}{2h}$

$=\frac{gh}{2V^2\cos^2\theta}.$

(d) From the diagram,

$r=h\tan\alpha+h\tan\beta=h(\tan\alpha+\tan\beta)$

$w=r-2h\tan\alpha=h(\tan\alpha+\tan\beta)-2h\tan\alpha$

$=h(\tan\alpha-\tan\beta).$

(e) $\tan\phi=\frac{\dot{y}}{\dot{x}}=\frac{V\sin\theta-gt_1}{V\cos\theta}$

$=\tan\theta-\frac{g}{V\cos\theta}\frac{h}{V\cos\theta\tan\alpha}$

$=\tan\theta-\frac{gh}{V^2\cos^2\theta}\frac{1}{\tan\alpha}$

$=\tan\theta-2\tan\beta$

$=\tan\alpha+\tan\beta-2\tan\beta$

$=\tan\alpha-\tan\beta.$

(f) $\frac{w}{r}=\frac{h(\tan\alpha-\tan\beta)}{h(\tan\alpha+\tan\beta)}$

$=\frac{\tan\alpha-\tan\beta}{\tan\alpha+\tan\beta}=\frac{\tan\phi}{\tan\theta}.$

$\therefore\frac{w}{\tan\phi}=\frac{r}{\tan\theta}.$